Electromagnetic Cavities

Resonant Cavities

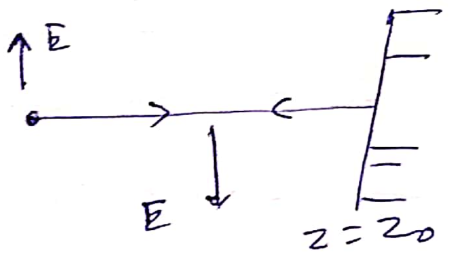
Purpose of wave guides → to transmit electromagnetic energy from one point to another

Resonator → An energy storage device equivalent to a resonant circuit element.

phenomena → Similar to standing wave patterns on a string occur in electromagnetic resonance

Consider a plane wave with components E_x and H_y → travelling in the z direction.

Perfect conductor → placed into the half infinite space $z > z_0$



Since the wave incident on the conductor turns back, the total electric field is given by

$$E_x = A_i e^{i(kz-\omega t)} + A_r e^{-i(kz+\omega t)}$$

This has to satisfy the boundary conditions

$$E_x = 0 \text{ at } z = z_0$$

$$\Rightarrow A_i e^{i(kz_0 - \omega t)} + A_r e^{-i(kz_0 + \omega t)} = 0 \quad \text{i.e.}$$

$$A_r = A_i e^{2ikz_0}$$

$$\Rightarrow E_x = A_i \left\{ e^{i(kz - \omega t)} - e^{2ikz_0} e^{-i(kz + \omega t)} \right\}$$

$$= A_i e^{-i\omega t} \left\{ e^{ikz} - e^{2ikz_0} e^{-ikz} \right\}$$

$$= A_i e^{-i\omega t} e^{ikz_0} \left\{ e^{ik(z - z_0)} - e^{-ik(z - z_0)} \right\}$$

$$= 2i A_i e^{-i\omega t} e^{ikz_0} \sin k(z - z_0) \quad \text{——— (1)}$$

Standing wave.

The standing wave pattern does not undergo any change → if an infinitely conducting plate is placed parallel to the first at a position of a node → where the electric field is zero.

The two plates → constitute a resonator in which the electromagnetic energy bounces between the plates

Consider a closed box formed by placing end faces on a rectangular wave guide. We assume that the end surfaces are plane & perpendicular

Because of the reflection at the end faces → the waves in the cavity are standing waves and not progressive.

The cavity resonator → resonate at a frequency at which the length of the cavity is an integral multiple of the half wavelength measured in the waveguide.

The electric field components are of the form

$$\left.\begin{aligned} E_x &= E_1 \cos(k_1 x) \sin(k_2 y) \sin(k_3 z) e^{-i\omega t} \\ E_y &= E_2 \sin(k_1 x) \cos(k_2 y) \sin(k_3 z) e^{-i\omega t} \\ E_z &= E_3 \sin(k_1 x) \sin(k_2 y) \cos(k_3 z) e^{-i\omega t} \end{aligned}\right\} \quad \text{--- (2)}$$

In order → the boundary conditions be satisfied → it is necessary that k_1, k_2, k_3 have the values

$$k_1 = \frac{l\pi}{a}, \quad k_2 = \frac{m\pi}{b}, \quad k_3 = \frac{n\pi}{c} \quad \text{--- (3)}$$

where a, b, c are the dimensions of the box and l, m, n → are integers.

Substituting any component in the appropriate wave-eq^n shows that the fields given by eq^n (3) be acceptable.

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(48)

The free space wave number has to satisfy the condition

$$k^2 = \frac{\omega^2}{c^2} = \pi^2\left(\frac{l^2}{a^2}+\frac{m^2}{b^2}+\frac{n^2}{c^2}\right) \quad \text{--- (4)}$$

infinite number of resonant frequencies → infinite number of modes of the cavity corresponding to the different values of l, m, n.

The magnetic field components can be found from Maxwell's eqn

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

if $l=0$, $m=1$ and ~~m=1~~ $n=1$ → electric field is transverse to the direction of propagation found from eqn (2) and (3)

↓

TE_{011} mode. Besides TE_{lmn} modes there are also other modes possible in the cavity.

TM_{lmn} modes → magnetic field is transverse to to the direction of propagation.

For same cavity → TE_{lmn}, TM_{lmn} → waves occur at the same frequency.

At a particular resonant frequency → standing wave in the cavity is the sum of two resonating waves, the TE mode and TM mode.

Theory of black-body radiation → factor 2 appears in the density of states function

For a definite field configuration
↳ the cavities have discrete resonant frequencies.

The right sort of fields will not be built up unless the exciting frequency is equal to the resonance frequency. However, appreciable excitation occurs over a narrow band of frequencies around the resonant frequency

Smearing out of the sharp frequency of oscillator
→ because of dissipation of energy in the cavity walls.

The measure of these ~~l~~ losses → expressed by "Q" of the cavity

$$Q = \omega \times \frac{\text{Energy stored in the cavity}}{\text{Energy lost per cycle to the walls}} \quad \text{—(5)}$$

Power loss → can be estimated by computing the time average of the Poynting vector into the wall at the surface

$$\langle N \rangle = \frac{1}{2} \mathrm{Re}\,(\underbrace{E_{\parallel} \times H_{\parallel}}_{\text{tangential components}})$$

Cavities → often used as frequency meters.

Cylindrical cavities → made more accurately than rectangular cavities.

Cavities → also used in exp. ~~where~~ where high microwave fields are required → ESR exp.

Electron Spin Resonance

Resonators → Open structures

↳ Two reflectors facing each other

↳ may be flat or spherical

↳ useful at optical wavelengths

↳ Used in Lasers